

23p.

NGS-88855

X63 15785  
(NASA TMX 50611)  
Code 2A

LIFT CONTROL IN MAGNETOHYDRODYNAMICS

By Adolf Busemann

6 refs

NASA Langley Research Center  
Langley Field, Va., U.S.A.

Presented at the International Symposium on Applications  
of the Theory of Functions in Continuum Mechanics

Tbilisi, Russia  
September 18-24, 1963

~~Available to NASA Offices and  
NASA Centers Only.~~

**CASE FILE COPY**

# LIFT CONTROL IN MAGNETOHYDRODYNAMICS

By Adolf Busemann

NASA Langley Research Center

## INTRODUCTION

In the first paper for the Hydrodynamical Section of this Symposium about Analytic Functions in Continuum Mechanics I like to return to an early and famous application in classical hydrodynamics made by W. M. Kutta and N. E. Joukowski while studying the lift force created in two-dimensional flow by a perfect fluid. The doubly connected region around a single profile in the infinite plane allows for a circulatory motion, the intensity of which is not determined by immediate boundary conditions. Nevertheless both these authors selected the same condition of smooth flow at the sharp trailing edge of either the flat plate (Kutta) or the special series of sharply edged Joukowski profiles as the proper lift-controlling element for the perfect flow, generally known as the "Kutta-Joukowski Condition." The smooth flow, however plausible it may appear, can only be a conjecture within the theory of perfect fluids and the rigorous proof resulting from flow problems with vanishing viscosity was added soon after by the boundary-layer theory of L. Prandtl.

A similar problem is found in the modern magnetohydrodynamics of two dimensions with a fluid having two properties to perfection, that is, vanishing viscosity paired with vanishing electrical resistivity while an external magnetic field is spread throughout the whole flow field. Regarding the distribution of two fields, the velocity and the magnetic vectors, it may appear that there are also two circulations around the profile undetermined, but, as is easy to conceive, these two circulations are fortunately tied to each other and only one combined circulation remains open and is responsible for any side force or lift. Unfortunately the simplified treatment adopted in this paper is a linearized approach to any lift-controlling elements of the body shape. Therefore, the ease to find a large variety of illustrative examples must be relied upon to hope that the linearized method may illuminate the problem sufficiently to disclose the magnetohydrodynamical generalization of the Kutta-Joukowski as a similarly plausible conjecture.

In this paper, [a short derivation of the differential equations and finally their specialization to two-dimensional flow will be given. Immediately thereafter, the general solution will be produced just as in my von Kármán 80th Anniversary Lecture, 1961.] It is exactly this step that led me to believe that its discussion at this Symposium might be appropriate. After showing a couple of examples in figures and pointing out their essential features, some conclusions about the lift-controlling elements will be added.

# SYMBOLS

B,b	magnetic field and disturbance field measured as Alfvén speed (actually $B/\sqrt{\rho\mu}$ )
c	profile chord
E	electric field
F,f	forces and volume forces
i	imaginary unit
J,j	electric current and current density
L	lift
p	pressure (divided by density)
$Q(z, \bar{z}, t)$	function symbol for general solution
q	complex conjugate to analytic pressure
S	magnetic source strength
t	time
V,v	velocity vector and velocity disturbance
x,y	coordinates of flow plane
$z, \bar{z}$	$x + iy, x - iy$ complex radius vector
$\beta$	angle of attack
$\Gamma$	circulation
$\kappa$	resistivity $1/(\sigma\mu)$
$\mu$	permeability (generally suppressed)
$\nu$	kinematic viscosity
$\pi$	$p + iq$ complex pressure function
$\rho$	fluid density (generally suppressed)
$\sigma$	electric conductivity
$\phi$	magnetic flux function

$\psi$  stream function

Subscripts:

h hydrodynamic

m magnetic

## DIFFERENTIAL EQUATIONS

For the sole purpose of showing the changes of the Kutta-Joukowski lift problem by electromagnetic interaction, it is permissible to take the name magnetohydrodynamics literally by assuming an incompressible fluid with a constant electric conductivity value regarded as a scalar quantity. The essential and, in the end, the only remaining variables of the flow field are the two vectors: the magnetic induction  $B$  and the fluid velocity  $V$ . Both these vector fields have lost part of their generality by the absence of divergences:

$$\operatorname{div} B = 0 \quad (1)$$

$$\operatorname{div} V = 0 \quad (2)$$

the first equation being a natural property by selecting the magnetic inductions as the representative of the magnetic field strength, the second equation being a mere convenience resulting from the assumption of incompressible fluid.

The square of both field vectors  $\frac{1}{2\mu}B^2$  and  $\frac{1}{2}\rho V^2$  indicate in connection with the proper constants in a magnetically permeable and incompressible fluid magnetic pressures and hydrodynamic pressures. This property makes them commensurable in the form of the Alfvén speed  $B/\sqrt{\rho\mu}$  compared with the velocity  $V$ . To simplify the equations, the density  $\rho$  and the permeability  $\mu$  are set equal to unity. Thus, the kinematic viscosity is the proper dissipative constant in this system, while for the kinematic "resistivity" of electrical currents, the reciprocal conductivity has to be divided by the permeability  $\mu$ :

$$\kappa = \frac{1}{\sigma\mu} \quad (3)$$

The influence on the hydrodynamic behavior of the movement across a magnetic field is caused by the so-called Lorentz forces of electrodynamics and their value  $f$  per unit volume is dependent on the flow of an electric current with the area intensity  $j$  as follows:

$$f = j \times B \quad (4)$$

however, any current reveals itself by the curling magnetic field around it according to the equations:

$$\mathbf{j} = \text{curl } \mathbf{B} \quad (5)$$

In combining these two expressions, the Lorentz forces immediately show how they are anchored in the magnetic field stress tensor  $B_i B_k - \frac{1}{2} \delta_{ik} B^2$ :

$$\mathbf{f} = (\text{curl } \mathbf{B}) \times \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{B} - \frac{1}{2} \nabla B^2 \quad (6)$$

These forces enter the hydrodynamic equation as follows:

$$\mathbf{f} - \text{grad } p_h = \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} - \nu \nabla^2 \mathbf{V} \quad (7)$$

The magnetic pressure  $p_m$ , the gradient of which is visible in the last term of equation (6), offers a summation with the static pressure  $p_h$  to the combined pressure  $p$  as indicated:

$$p = p_h + p_m = p_h + \frac{1}{2} B^2 \quad (8)$$

the gradient of which is simply:

$$-\text{grad } p = \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla - \nu \nabla^2 \right) \mathbf{V} - (\mathbf{B} \cdot \nabla) \mathbf{B} \quad (9)$$

In the absence of imposed electric fields all currents  $\mathbf{j}$  are caused by induction in the fluid moving across the magnetic field or vice versa. Closed currents are already assured when firstly the curl of the induced electric field  $\mathbf{E}$  is used:

$$\text{curl } \mathbf{E} = \text{curl}(\mathbf{V} \times \mathbf{B}) - \frac{\partial \mathbf{B}}{\partial t} \quad (10)$$

and secondly the electric resistivity  $\kappa$  of the fluid is a constant:

$$\text{curl } \mathbf{E} = \kappa \text{ curl } \mathbf{j} = \text{curl}(\mathbf{B} \times \mathbf{B}) - \frac{\partial \mathbf{B}}{\partial t} \quad (11)$$

Avoiding again the explicit appearance of the currents or the electric field, a relation between the magnetic field and the velocity field results out of equation (11) combined with (5):

$$\kappa \operatorname{curl} \operatorname{curl} B = \operatorname{curl}(B \times B) - \frac{\partial B}{\partial t} \quad (12)$$

After the usual transformation of all double cross products into two terms of dot products, while considering that all divergences vanish according to equations (1) and (2), the same relation can be written

$$-\kappa \nabla^2 B = B \cdot \nabla - V \cdot \nabla B - \frac{\partial B}{\partial t} \quad (13)$$

All differential operations except one are carried out on the magnetic field. Two operator packages in parentheses show this fact more clearly:

$$\left( \frac{\partial}{\partial t} + V \cdot \nabla - \kappa \nabla^2 \right) B - (B \cdot \nabla) V = 0 \quad (14)$$

This final form of the electromagnetic equation has a strange resemblance to the final hydrodynamic equation (9). The third terms of the large operators differ only by their diffusion coefficients which are the viscosity  $\nu$  in hydrodynamics and the resistivity  $\kappa$  in electromagnetism. If such a difference, which of course disappears for perfect fluids, should cause too much trouble, there is always the possibility of finding a first solution for the fluid with  $\nu = \kappa$  or the "equi-dissipative" fluid.

#### LINEARIZATION

Any differential operator parenthesis used in the final equations (9) and (14) contains a term having one of the unknown field vectors as a coefficient, as a reminder of the nonlinearity of both relations. In these terms lies the actual difficulty of solving the complete problem by finding particular solutions. However, such an approach can be used after linearization of the differential equations for small disturbance flows. Since all terms in the operators represent partial differentiations with respect to space or time, an undisturbed pattern consisting of a parallel flow and a parallel magnetic field - though in different directions, if preferred - vanishes identically if operated upon regardless of any unknown field used inside the operators. Under these circumstances the linearization requires the unknown disturbance fields to be always the object of the operations, while the field inside the operators may as well be the unadulterated undisturbed one.

If we designate with the former letters  $V$  and  $B$  the main fields and use for the disturbances  $v$  and  $b$  as small additions to the parallel fields, the operators in equations (9) and (14) are practically differential operators with fixed coefficients in which the field vectors  $V$  and  $B$  are considered to be the values  $V_\infty$  and  $B_\infty$  sufficiently far away from the disturbing body, without being

explicitly designated as such by a subscript. This linearization allows the operators to be treated as commuting factors as long as they operate on the same field  $v$  or  $b$ . The new form of the equations (9) and (14) due to linearization is as follows:

$$\left(\frac{\partial}{\partial t} + v \cdot \nabla - \nu \nabla^2\right)v - (B \cdot \nabla)b = -\text{grad } p \quad (15)$$

$$\left(\frac{\partial}{\partial t} + v \cdot \nabla - \kappa \nabla^2\right)b - (B \cdot \nabla)v = 0 \quad (16)$$

The early equations (1) and (2) adapted to the new symbols for the disturbance fields are:

$$\text{div } b = 0 \quad (1a)$$

$$\text{div } v = 0 \quad (2a)$$

It is obvious that these four equations, half of which are scalar equations while the other half are vectors with the disturbance function  $p$ , have enough information to allow the unknown fields to be determined.

#### REDUCTION TO TWO DIMENSIONS

Since the best hope of finding solutions is given in the two-dimensional flow, the further treatment of these equations is carried out in the complex plane in accord with W. M. Kutta and N. E. Joukowski. The first convenience resulting from the reduction to two dimensions is the existence of a scalar flux function  $\phi$  for the magnetic field  $b$  and a scalar stream function  $\psi$  for the velocity field as integrals of the equations (1a) and (2a). In the complex plane  $z = x + iy$  with  $\bar{z} = x - iy$  the nabla operator is known to be

$$\nabla = 2 \frac{\partial}{\partial \bar{z}}$$

of any function written in the variables  $z$  and  $\bar{z}$ . While a gradient can immediately be written in nabla, the relation between flux functions or stream functions and the vector field parallel to them only requires a multiplier  $\pm i$  to perform a rotation about  $\pm 90^\circ$ . The vector fields are, therefore, easily related to their flux integrals:

$$b = -i 2 \frac{\partial}{\partial \bar{z}} \phi \quad (17)$$

$$v = -i2\frac{\partial}{\partial \bar{z}}\psi \quad (18)$$

While the equation (16) may be raised to the flux function level as follows:

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla - \kappa \nabla^2\right)\phi - (B \cdot \nabla)\psi = 0 \quad (16a)$$

the right-hand side of equation (15) is not quite suited to this treatment. But the vanishing divergences of  $v$  and  $b$  spotted on the left-hand side discloses  $p$  to be an analytic function in the whole domain outside the body:

$$\text{div grad } p = 0 \quad (19)$$

If this fact is appreciated, the pressure  $p$  must be the real part of a complex "pressure" function  $\pi$ :

$$p + iq = \pi(z, \bar{z}, t) \quad (19a)$$

Since  $p$  and its complex conjugate  $q$  carry the same information, it is simple to conceive that  $i \text{ grad } q$  can replace  $\text{grad } p$  as a result of conformal mapping. The raising of equation (15) to flux level is consequently performed by exchange of  $p$  with  $q$ :

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla - \nu \nabla^2\right)\psi - (B \cdot \nabla)\phi = q \quad (15a)$$

with the understanding that  $q$  is an analytic function:

$$\text{div grad } q = \nabla^2 q = 4 \frac{\partial^2}{\partial z \partial \bar{z}} q = 0 \quad (19b)$$

The equations (15a) and (16a) represent two scalar equations for two unknown scalar functions  $\phi$  and  $\psi$  while  $q$  may already serve as an arbitrary analytic function encountered during the integration.

#### GENERAL SOLUTION

The equation (16a) can be integrated by a "potential"  $Q$  because of the commuting of all differential operators used:

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla - \kappa \nabla^2\right)Q = \psi \quad (20)$$

$$(B \cdot \nabla)Q = \phi \quad (21)$$

and the new function  $Q(z, \bar{z}, t)$  gets its restriction from the equation (15a) as follows:

$$\left( \left( \frac{\partial}{\partial t} + V \cdot \nabla - \nu \nabla^2 \right) \left( \frac{\partial}{\partial t} + V \cdot \nabla - \kappa \nabla^2 \right) - (B \cdot \nabla)^2 \right) Q = q \quad (22)$$

While the second summand is always a square, the first summand would also be a square of a single operator in the case of perfect flow without any dissipations. If however dissipations are considered essential, only a fluid with equal dissipation coefficients  $\kappa = \nu$ , the "equi-dissipative" fluid, allows to set:

$$\begin{aligned} & \left( \left( \frac{\partial}{\partial t} + V \cdot \nabla - \nu \nabla^2 \right)^2 - (B \cdot \nabla)^2 \right) Q \\ &= \left( \frac{\partial}{\partial t} + (V - B) \cdot \nabla - \nu \nabla^2 \right) \left( \frac{\partial}{\partial t} + (V + B) \cdot \nabla - \nu \nabla^2 \right) Q \\ &= q \end{aligned} \quad (22a)$$

According to the character of the "conjugate to the pressure"  $q$  of equation (19b), the new function  $Q$  may be written as the operand of four independent operators as follows:

$$\left( \frac{\partial}{\partial t} + (V - B) \cdot \nabla - \nu \nabla^2 \right) \left( \frac{\partial}{\partial t} + (V + B) \cdot \nabla - \nu \nabla^2 \right) \left( \frac{\partial}{\partial z} \right) \left( \frac{\partial}{\partial \bar{z}} \right) Q = 0 \quad (23)$$

It promises an integral of four additive contributions:

$$Q = Q_1 + Q_2 + Q_3 + Q_4 \quad (24)$$

With a specific adaptation to one operator in each part:

$$\left( \frac{\partial}{\partial t} + (V - B) \cdot \nabla - \nu \nabla^2 \right) Q_1 = 0 \quad (24a)$$

$$\left( \frac{\partial}{\partial t} + (V + B) \cdot \nabla - \nu \nabla^2 \right) Q_2 = 0 \quad (24b)$$

$$2 \frac{\partial}{\partial z} Q_3 = 0 \quad (24c)$$

$$2 \frac{\partial}{\partial \bar{z}} Q_4 = 0 \quad (24d)$$

Going back to the original unknowns is necessary for the boundary conditions. The equations (20) and (21) lead back one step to the flux and stream functions, while equations (17) and (18) add the second step toward the magnetic fields and velocity fields themselves. The application of (20) and (21) on parts of  $Q$  discloses their specific meaning:

$$\left(\frac{\partial}{\partial t} + (V - B) \cdot \nabla - \nu \nabla^2\right) Q_1 = \psi_1 - \phi_1 = 0 \quad (25a)$$

$$\left(\frac{\partial}{\partial t} + (V + B) \cdot \nabla - \nu \nabla^2\right) Q_2 = \psi_2 + \phi_2 = 0 \quad (25b)$$

Since the only disturbances possible in fields with vanishing divergences are vortices, the solution  $Q_1$  indicates co-rotating vorticity of both velocity and magnetic field travelling with the speed  $V - B$  except for dissipation. In the same manner  $Q_2$  is a contribution by contra-rotating vorticity in both fields travelling with the speed  $V + B$ . When the steady flow state is reached, the vorticity along any line  $V - B$  or  $V + B$  is uniform except for dissipation and the flow pattern resembles very much the supersonic two-dimensional flow (fig. 1). The time dependent terms in the above equations are very useful in indicating the directions in which to employ the two families of waves starting at the body contour. Playing with this portion of the solution reveals only (slender) bodies without angle of attack to have neither a sink nor a source of magnetic flux inside the body. No pressures are created and the lift is always zero. But this is only the timid approach to studying lift. Pressures result from the solutions  $Q_3$  and  $Q_4$  representing complex analytical functions of  $\bar{z}$  respectively of  $z$ .

While reducing  $Q_3$  and  $Q_4$  to the flux and stream functions according to equations (21) and (20), it is always allowable to omit the last term in the operator of equation (20), since analytic functions have no dissipative contributions:

$$\phi_3 = B \cdot \nabla Q_3 \quad \psi_3 = \left(\frac{\partial}{\partial t} + V \cdot \nabla\right) Q_3 \quad (25c)$$

$$\phi_4 = B \cdot \nabla Q_4 \quad \psi_4 = \left(\frac{\partial}{\partial t} + V \cdot \nabla\right) Q_4 \quad (25d)$$

For the final steady flow state the relations between the flux and the stream disturbance are even closer without the first terms for the stream function:

$$\psi_3 = V \cdot \nabla Q_3 = \left(V \frac{\partial}{\partial z} + \bar{V} \frac{\partial}{\partial \bar{z}}\right) Q_3(\bar{z}) = \bar{V} Q_3' \quad (26a)$$

$$\phi_3 = \mathbf{B} \cdot \nabla Q_3 = \left( B \frac{\partial}{\partial z} + \bar{B} \frac{\partial}{\partial \bar{z}} \right) Q_3(\bar{z}) = \bar{B} Q_3' \quad (26b)$$

In the steady state the complex potentials are simply proportional

$$\phi_3 = \frac{\bar{B}}{V} \psi_3 \quad (26)$$

The velocities derived from these potentials have the same ratio

$$b_{34} = \frac{\bar{B}}{V} v_{34} \quad (27)$$

The singularities from these disturbances are normally vortices hidden inside the body and outside of the body represent the circulatory motion or magnetic field responsible for the lift. Both the flow and the magnetic field are not allowed to show sinks or sources, but a complex multiplier on one potential as in equation (26) intermixes circulations and sources. The result shown in figure 2 is that the familiar circulation of the velocity field would change into a circulation plus a sink for the magnetic field.

If we reject the solutions  $Q_3$  and  $Q_4$  except for parallel fields  $V$  and  $B$ , because of the unaccountable sink strength, the timid approach to studying lift would be perfect. The real conclusion is to combine the flux surplus in any solution  $Q_1$  and  $Q_2$  created by finite angle of attack with the flux sink of  $Q_3$  and  $Q_4$ , and to get lift, or at least circulation controlled by the angle of attack. Figures 3 and 4 are prepared to help in finding the flux surplus  $\Delta\phi_{12}$  according to angle of attack. With the help of the fishbone pattern inside the body of figure 1 in figure 3 starting at the leading edge of the body, the book-keeping of the magnetic flux for any body shape is simple. At zero angle of attack the surplus is zero, the horizontal to vertical size relations are taken from the vector diagram  $V, B$  to demonstrate that  $2V$  horizontally corresponds to  $B_y$  vertically (fig. 4). The flux through any horizontal line unit is also  $B_y$ . The result for a chord  $c$  and an angle  $\beta$  downward amounts to:

$$\Delta\phi_{12} = c B_y \beta \frac{2V}{B_y} = 2Vc\beta \quad (28)$$

With the same coordinates lined with  $x$  parallel to  $V$  the ratio  $\bar{B}/V$  is changing the velocity circulation  $\Gamma$  into the magnetic circulation  $J$  and the magnetic source strength  $S$ :

$$J + iS = \frac{\bar{B}}{V} \Gamma = \frac{B_x - iB_y}{V} \Gamma \quad (29)$$

A source strength compensating the surplus of flux in equation (28) leads to the circulation  $\Gamma$ :

$$\Gamma = \frac{2V^2 c}{B_y} \beta \quad (30)$$

The force created by circulation in the combined hydromagnetic field is composed of the lifting force  $iV\Gamma$  and the Lorentz force  $-iBJ$ . But instead of the expected simple current  $J$  indicating the magnetic circulation around the body, the complex multiplication (26) creates  $J = \Gamma \frac{\bar{B}}{V}$ . The result is:

$$F = iV\Gamma - iB\left(\Gamma \frac{\bar{B}}{V}\right) = iV\Gamma \left(1 - \frac{B\bar{B}}{VV}\right) \quad (31)$$

The resulting force  $F$  is indeed a side force in the manner of the hydrodynamic lift, but it is reduced by the factor  $1 - (B/V)^2$  which indicates vanishing lift at movements with Alfvén speed and a lift reversal for sub-Alfvénic speeds that is to say too strong magnetic fields.

Concerning the generalized Kutta-Joukowski condition, the case of nonaligned fields  $V$  and  $B$  seems quite like supersonic flow, where the leading and the trailing edge may help to define the lift with precision by being sharpened, while the angle of attack is in complete control of the circulation.

#### MOVEMENT PARALLEL TO MAGNETIC FIELD

While the Kutta-Joukowski problem is seen to disappear as such, when the main magnetic field is under a finite angle with respect to the flow, the aligned motion with (or against) the direction of the magnetic lines is an exception, in which the two vorticity strips starting at the body surface of figure 1 degenerate to a single streamline behind or in front of the body. The potential flow disturbance according to the combined solutions  $Q_3$  and  $Q_4$  makes the magnetic lines coincide with the steady-state streamlines. Thus the problem in the large is exactly the familiar hydrodynamic problem and after it is solved, its streamlines are also used as magnetic lines. Only the vanishing imperfections by viscosity and by resistivity must be investigated for differences in the wake regions.

Mathematically the alinement of both the undisturbed velocity  $V$  and the magnetic field  $B$  with the x-axis is of not much immediate consequence. The general solution of equation (22) still has too many different terms:

$$\left(\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x} - v\nabla^2\right)\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x} - \kappa\nabla^2\right) - B^2 \frac{\partial^2}{\partial x^2}\right)Q = q \quad (22b)$$

If, however, the steady flow is studied with a semi-perfect fluid in which either the electric resistivity or the viscosity is zero, one factor  $\frac{\partial}{\partial x}$  will be found separable:

$$\left(\frac{v^2 - B^2}{V} \frac{\partial}{\partial x} - \nu \nabla^2\right) \left(V \frac{\partial}{\partial x}\right) Q = q \quad \text{for } \kappa = 0 \quad (22c)$$

or

$$\left(\frac{v^2 - B^2}{V} \frac{\partial}{\partial x} - \kappa \nabla^2\right) \left(V \frac{\partial}{\partial x}\right) Q = q \quad \text{for } \nu = 0 \quad (22d)$$

Splitting the first particular solutions  $Q_1$  and  $Q_2$  between these factorized operators as it is done for the equi-dissipative fluid in (24a) and (24b), makes the second one  $Q_2$  uninteresting, while the first one  $Q_1$  has not  $V - B$  but a new convective velocity  $(v^2 - B^2)/V$  modifying the dissipation of vorticity. This new convective velocity joins the zero value of  $V - B$  but has the added factor  $(V + B)/V$  which doubles for flows with about Alfvén speed and grows unlimitedly with increasing  $B$ . Having no immediate check on the nonsteady build-up process, this new velocity resembles a "phase speed" compared to the "group speed"  $V - B$  of the equi-dissipative fluid. Both extremes of semi-perfect fluids differ with respect to each other, when the flux function and the stream function are derived from equations (20) and (21). For vanishing electrical resistivity (superconductivity) there results:

$$\varphi_1 = B \frac{\partial}{\partial x} Q_1 \quad \psi_1 = V \frac{\partial}{\partial x} Q_1$$

or

$$\varphi_1 = \frac{B}{V} \psi_1 \quad \text{for } \kappa = 0 \quad (23a)$$

while for nonviscous fluid it is found that

$$\left(\frac{(v^2 - B^2)}{V} \frac{\partial}{\partial x} - \kappa \nabla^2\right) Q_1 = \psi_1 - \frac{B}{V} \varphi_1 = 0$$

or

$$\varphi_1 = \frac{V}{B} \psi_1 \quad \text{for } \nu = 0 \quad (23b)$$

The result is that the superconductive fluid conserves the plain similarity  $\varphi : \psi = B : V$  in the boundary layer, while the nonviscous fluid reverses the ratio inside the boundary layer of moving vorticity.

Historically it was the superconductive fluid having only one boundary-layer solution  $Q_1$  of almost regular behavior on which the reversal of direction at  $B = V$  was first discovered, and it was this result that made the classical Kutta-Joukowski condition look ridiculous for strong magnetic fields  $B > V$  because of the forward wake.

## BOUNDARY CONDITIONS

While the linearization does not guarantee any disclosure of the lift controlling elements, the proper application of the boundary conditions for vanishing imperfections of the fluid on a large variety of examples is still the foundation for any hopes in that direction. The perfect flow itself determines the circulation by the angle of attack between sharp leading and trailing edges only when  $V$  and  $B$  have different directions. At complete alinement the circulation remains primarily uncontrolled.

For the sake of complete confidence in the linearized solutions of the magnetohydrodynamic problem the boundary conditions of the unalined case for the equi-dissipative fluid may be discussed first. Figure 1 shows that the upper and the lower side of the body produce one family of vorticity each already in the case of perfect flow, where the normal but not the tangential velocity at the body contour can be chosen. For being able to prescribe the tangential velocity independently, the second solution of either corotating or contrarotating vorticity is the only help available relying on the completeness of the solutions. Ordinarily the convective velocity of the second wave brands it as an incoming wave with no information to carry. But one should never underestimate the power of diffusion. Strong diffusion is able to make its way against any convective speed though, of course, with rather steep decay of the vorticity. If, therefore, the boundary of the body demands the second type of vorticity to be present, a thin layer of the second type is able to exist and it has a tangential velocity according to  $V - B_x$  or  $V + B_x$ , respectively (fig. 5). One part of the boundary-layer vorticity is able to move in upstream direction when  $V - B_x$  is negative. After running off at the trailing or even the leading edge of the body, the second vorticity type finally joins its own family as a narrow band of laces added to the former strip in figure 4.

Going now back to the case of alined velocity and magnetic fields, the original strips of perfect flow vorticity have folded into the body streamline but the boundary-layer vorticity and the diffusive spread of the edges of the original strips are still available (fig. 6). The equi-dissipative fluid has according to this picture two types of (a) corotating and (b) contrarotating vorticity available with the distinguishable convection speeds  $V - B$  and  $V + B$ . One of these types, the corotating one - if  $+V$  and  $+B$  are alined - is even able to move toward the leading edge of the body and beyond at sub-Alfvénic speeds.

The semi-perfect fluids have lost one of the diffusive properties that make ordinarily the occurrence of discontinuities impossible. Since discontinuities are now acceptable, the double boundary layer of the equi-dissipative fluid

changes to a single boundary layer (fig. 7) with the phase velocity  $(V^2 - B^2)/V$ . The reversal is again at Alfvén speed, but the leading edge is at sub-Alfvénic speeds the only one adjacent to a wake. Nonsteady shedding of vortices may still be expected at the trailing edge, but this process is not available in a factorized differential equation of  $Q$ .

## CONCLUSION

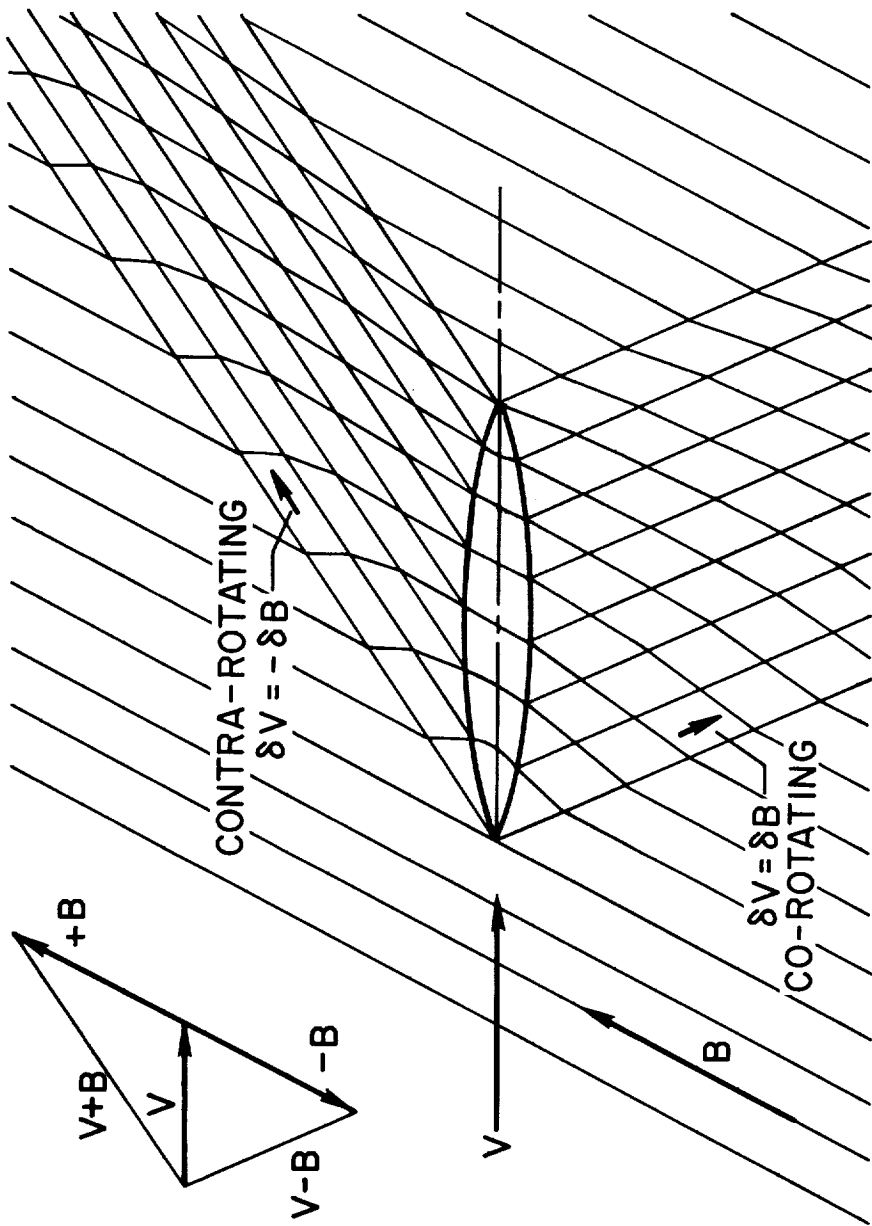
The two-dimensional magnetohydrodynamic flow past a profile is treated with linearization for only three special ratios between the imperfections caused by viscosity and electrical resistivity zero, one, and infinity. Under these circumstances the Kutta-Joukowski problem of lift control is not visible in all its details. If it were, not the generalized Kutta-Joukowski conjecture, but its boundary-layer proof would have been presented. In a sense this affair is again in the conjectural state and scientists have still the opportunity to risk their reputation with wrong guesses. Two results are unquestionable: Unalined velocity and magnetic fields have supersonic character, where a sharp leading and a sharp trailing edge control the lift by their angle of attack. The super-Alfvénic speed range of the aligned fields has the trailing edge as the lift controlling feature and the trailing edge should be sharp.

The remaining conjecture is about whether the leading and trailing edges should be sharp or not sharp at sub-Alfvénic speeds, where the wake or, at least, half the wake extends forward of the body. Some voices are for a complete reversal of all familiar incompressible or subsonic relations and ask for a sharp leading edge as the means of lift control. One word of caution may be added, that the former suction force on a sharp trailing edge supporting separation has changed to a compression force with a blunting tendency for materials of finite strength.

## REFERENCES

1. Sears, W. R., and Resler, E. L., Jr.: Theory of Thin Airfoils in Fluids of High Electrical Conductivity. Jour. of Fluid Mech. 5, Part 2, Feb. 1959.
2. Resler, E. L., Jr., and McCune, J. E.: Electromagnetic Interaction with Aerodynamic Flows. The Magnetohydrodynamics of Conducting Fluids, ed. by Daniel Bershader, Stanford Univ. Press, 1959.
3. Carrier, G. F., and Greenspan, H. P.: Magnetohydrodynamic Flow Past a Flat Plate. The Magnetodynamics of Conducting Fluids, ed. by Daniel Bershader, Stanford Univ. Press, 1959, and Jour. of Fluid Mech, 5, pp. 77-96, 1959.
4. Sears, W. R.: Studies of an Inviscid Boundary Layer of Magnetohydrodynamics. Tenth Internat. Appl. Mach. Congress, Stresa, Italy, 1960.
5. Busemann, A.: On the Kármán Vortex Street in Magnetofluid-Dynamics. 80th Anniversary Proceedings, IAS, 1962.
6. Busemann, A.: Relations Between Aerodynamics and Magnetohydrodynamics. NASA SP 25 of University Conference Chicago, November 1-3, 1962.





NASA

Figure 1.- Free vorticity.

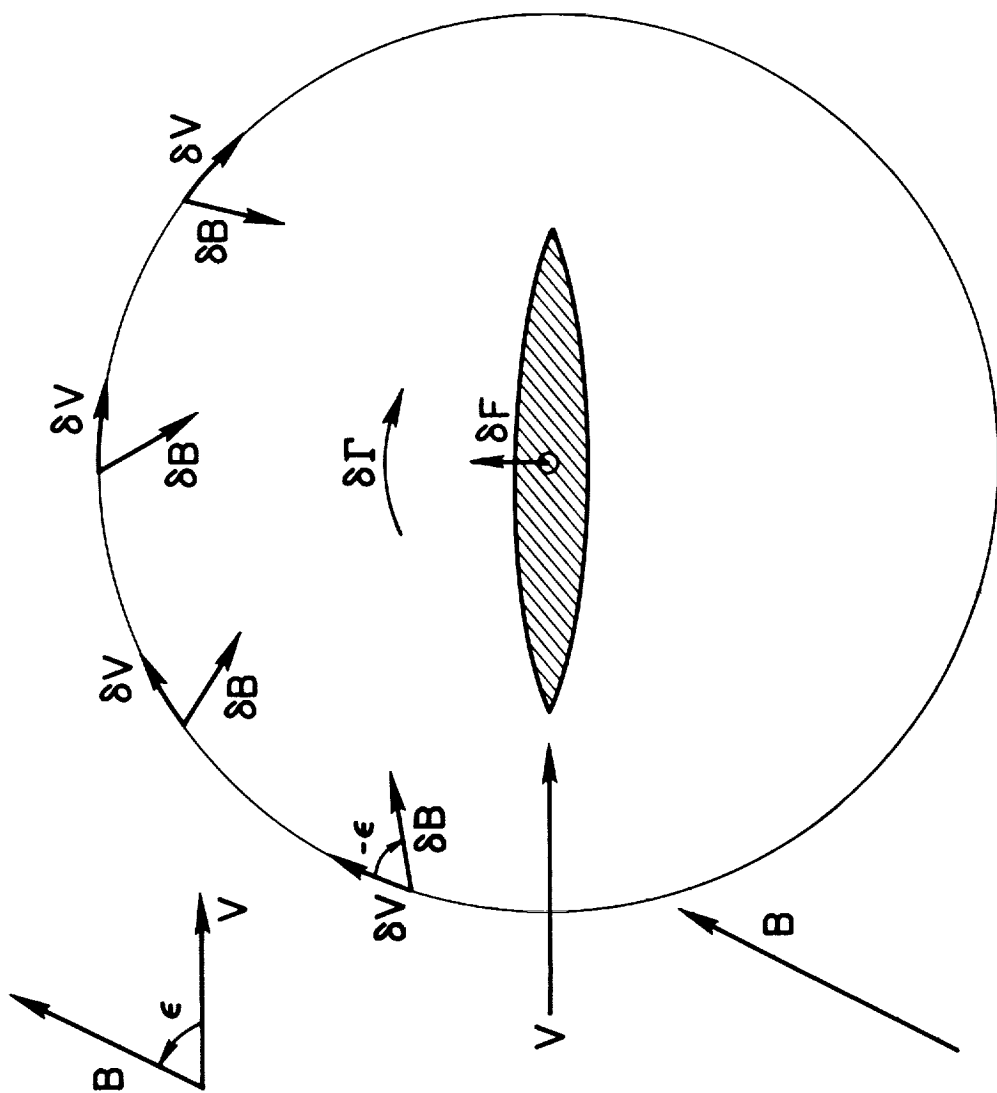
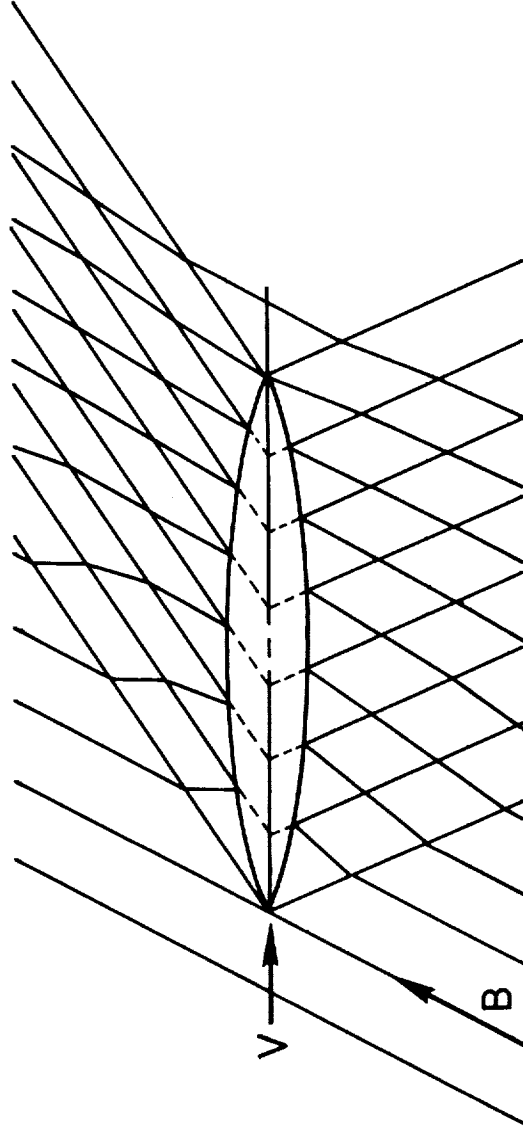
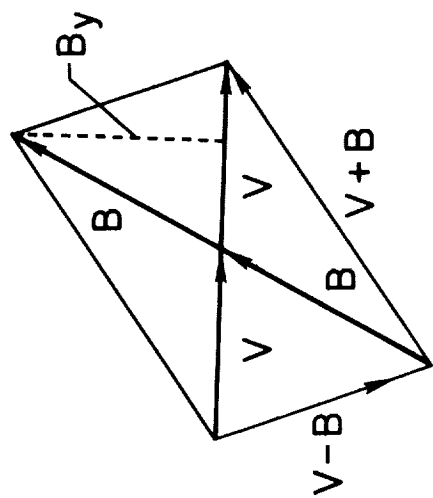


Figure 2.- Bound circulation.



NASA

Figure 3.- Magnetic flux balance.



$$\Delta\phi_{12} = c\beta B_y \frac{2V}{B_y}$$

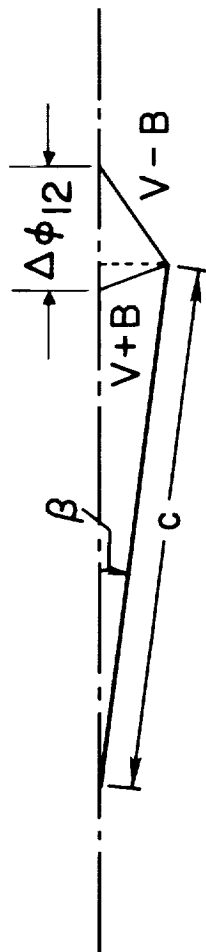
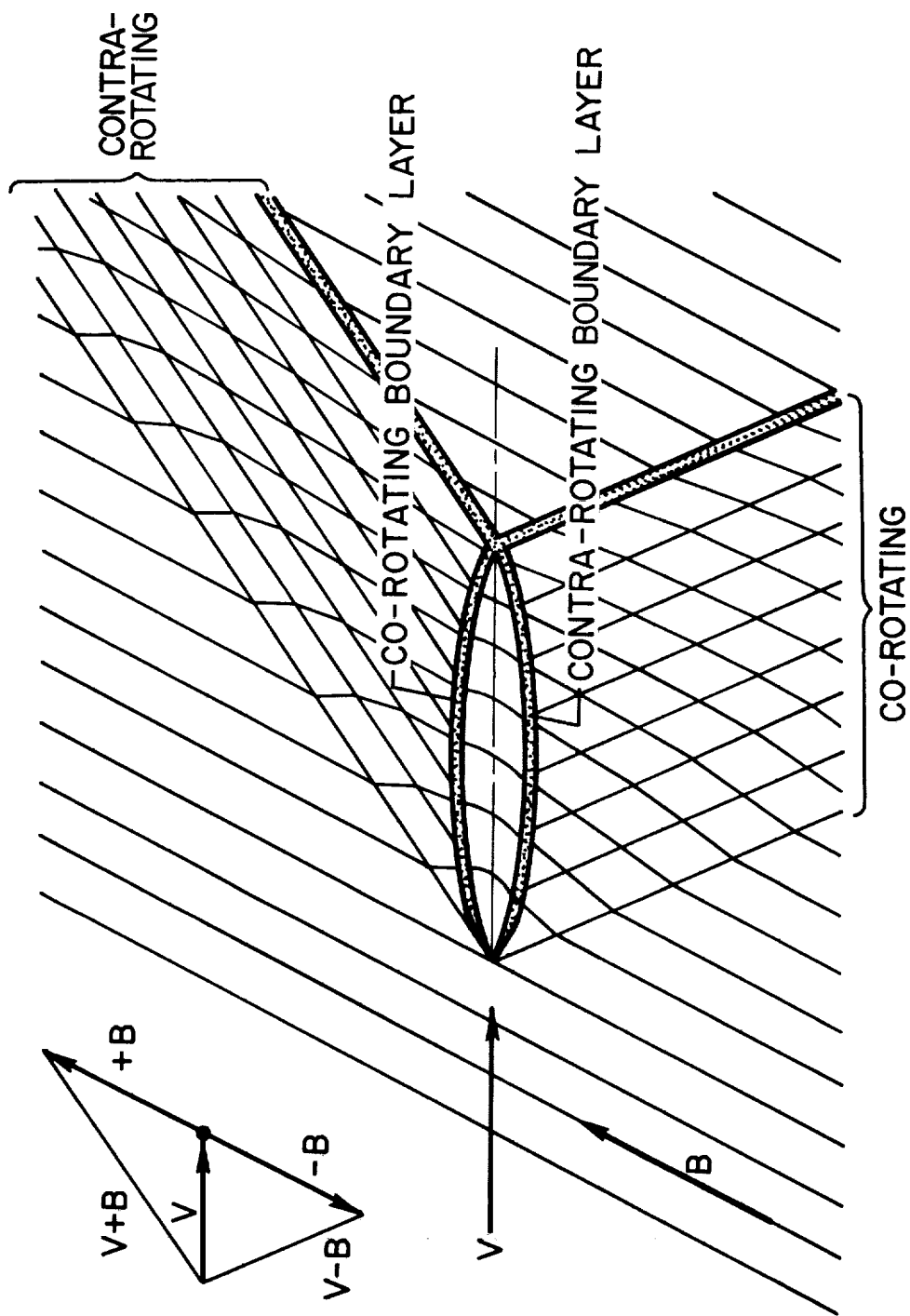


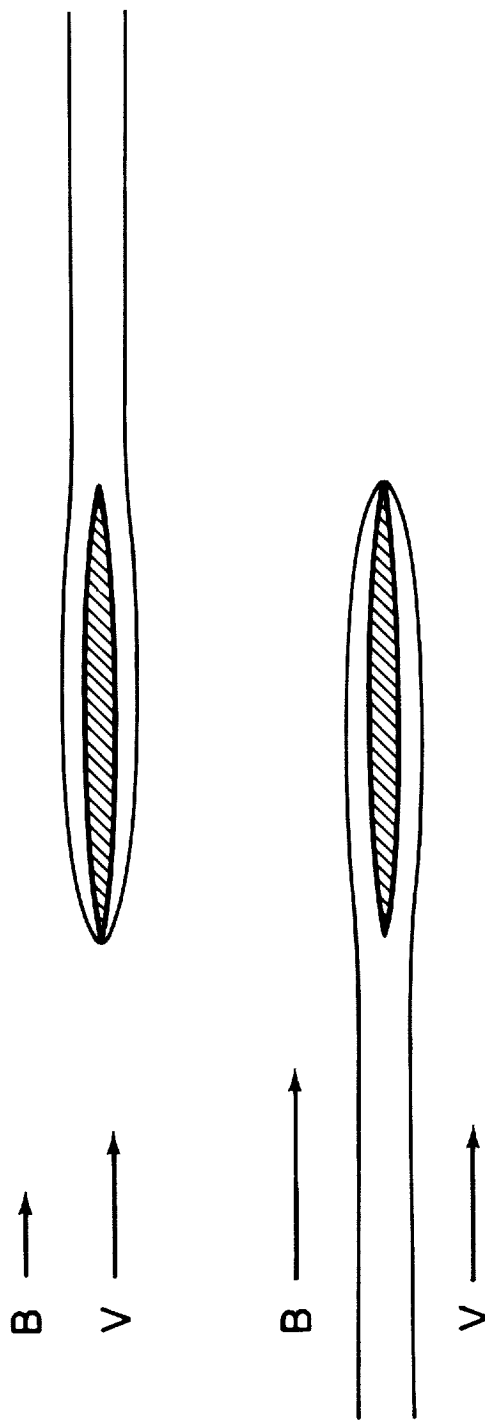
Figure 4.- Circulation control.



NASA

Figure 5.- Nonslip boundary condition.

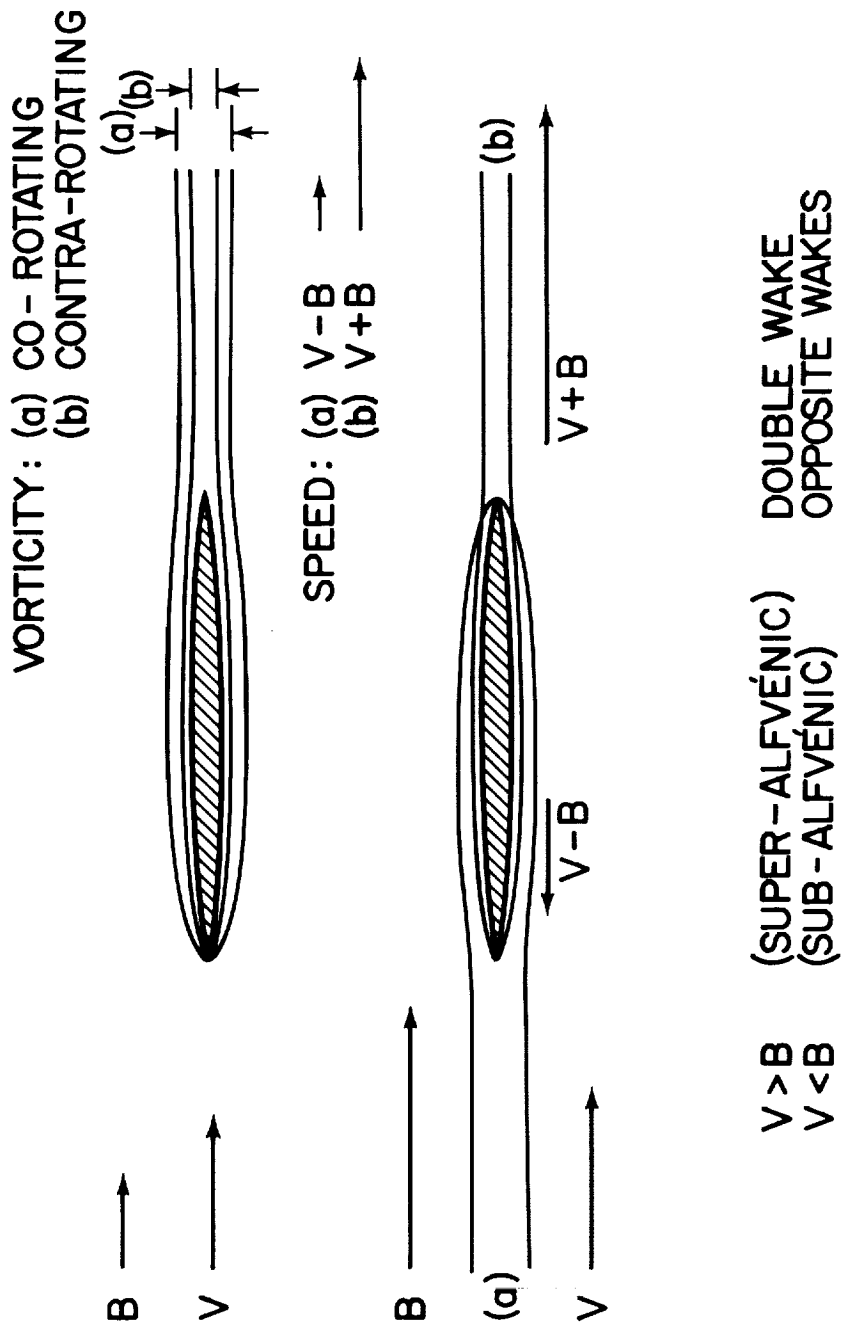
SEMI-PERFECT FLUID  
 $\kappa=0$  OR  $\nu=0$



$V > B$  (SUPER-ALFVÉNIC) WAKE BEHIND  
 $V < B$  (SUB-ALFVÉNIC) WAKE IN FRONT

NASA

Figure 6.- Boundary-layer reversal at  $B \parallel V$ .



NASA

Figure 7.- Isodissipative fluid at  $B \parallel V$ .

